One-sample t-test for means

Use case

We want to estimate the population mean with regard to a single, quantitave variable. We have a sample of size n and popluation mean $\bar{x}$. We don't know with certainty the population standard deviation, σ, so we estimate it using the sample standard deviation, s.

Preconditions

For this test to be reliable, one of the following conditions must hold:

1. The population is normally distributed.
2. The population is **roughly** normal, and our sample size is large.

Hypotheses

Null hypothesis, H_0:
$\mu = \mu_0$

Alternative hypothesis, H_a:
$\mu \neq / > / < \mu_0$

Test statistic

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

p-value

Where T is a random variable distributed according to the $t(k)$ distribution with degrees of freedom $k = n - 1$,

1. For $H_a : \mu < \mu_0$: $p = P(T \leq t)$
2. For $H_a : \mu > \mu_0$: $p = P(T \geq t)$
3. For $H_a : \mu \neq \mu_0$: $p = 2P(T \leq -|t|)$

Because the $t(k)$ distribution is slightly wider than the normal distribution, a p-value obtained using this method will be slightly larger than the p-value generated from a z-test.

p-value (Python)

1. For
   $H_a: \mu < \mu_0$: `p = stats.t.cdf(t, df=n-1)`
2. For
   $H_a: \mu > \mu_0$: `p = 1 - stats.t.cdf(t, df=n-1)`
3. For
   $H_a: \mu \neq \mu_0$: `p = 2 * stats.t.cdf(-abs(t), df=n-1)`

Associated confidence interval

$$\text{C\% confidence interval} = \bar{x} \pm t^{\star}\frac{s}{\sqrt{n}}$$
$$\text{choose } t^{\star} \text{ s.t. area on } t(n-1) \text{ distribution from } (-t^{\star}, t^{\star}) = \text{C}$$

Because our p-values are slightly larger than those from a z-test, our t-confidence interval will be slightly larger than a z-confidence interavl.

Minimum sample size to achieve margin of error:

$$m^{\star} = t^{\star}\frac{s}{\sqrt{n}} \iff n = (\frac{t^{\star}s}{m^{\star}})^2$$

Related tests

If you have access to the true value of the population standard deviation, a one-sample z-test for means is more appropriate.

If you want to compare means across two diferent independent populations, a two-sample t-test for means is more appropriate.

Considerations

A one-sample t-test conducted at confidence level
α will reject the null hypothesis if and only if the value corresponding to the null hypothesis,
μ_0, is completely outside of the
$C = 1 - \alpha$ t-confidence interval.

The p-value provided from a one-sample t-test will always be *slightly* larger than a corresponding z-test. This larger p-value reflects the increased uncertainty introduced by estimating the population standard deviation using
s.

Conducting a matched pairs t-test

A matched pairs t-test is conducted where the variable of interest is actually the difference between two variables for a given pair. After computing this quantity for each case, proceed with a one-sample t-test for means on this single difference variable.