One-sample z-test for means

Use case

We want to estimate the population mean with regard to a single, quantitave variable. We know with certainty the population standard deviation, σ. We have a sample of size n and popluation mean $\bar{x}$.

Preconditions

To estimate the population mean using the normal distribution, one of the following conditions must hold:

1. The population is normally distributed.
2. The sample size, n, is large enough for the Central Limit Theorem to take effect.

Hypotheses

Null hypothesis, H_0:
$\mu = \mu_0$

Alternative hypothesis, H_a:
$\mu \neq / > / < \mu_0$

Test statistic

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

p-value

Where Z is a standard normal random variable,

1. For $H_a : \mu < \mu_0$: $p = P(Z \leq z)$
2. For $H_a : \mu > \mu_0$: $p = P(Z \geq z)$
3. For $H_a : \mu \neq \mu_0$: $p = 2P(Z \leq -|z|)$

p-value (Python)

1. For $H_a : \mu < \mu_0$: `p = stats.norm.cdf(z)`
2. For

$H_a : \mu > \mu_0$: `p = 1 - stats.norm.cdf(z)`

3. For
   $H_a : \mu \neq \mu_0$: `p = 2 * stats.norm.cdf(-abs(z))`

Associated confidence interval

$$\text{C\% confidence interval} = \bar{x} \pm z^\star \frac{\sigma}{\sqrt{n}}$$
$$\text{choose } z^\star \text{ s.t. area on standard normal distribution from } (-z^\star, z^\star) = \text{C}$$

Minimum sample size to achieve margin of error:

$$m^\star = z^\star \frac{\sigma}{\sqrt{n}} \iff n = (\frac{z^\star \sigma}{m^\star})^2$$

Related tests

If you lack access to the true value of the population standard deviation, a one-sample t-test for means is more appropriate.

If you want to compare means across two diferent independent populations, a two-sample z-test for means is more appropriate.

Considerations

A one-sample z-test conducted at confidence level
α will reject the null hypothesis if and only if the value corresponding to the null hypothesis,
μ_0, is completely outside of the
$C = 1 - \alpha$ confidence interval.